

There follows a Z specification of a program that derives randomness from some kind of seed. The Z specification was written by Rob Arthan after a request by Dave Topham.

Z specification

Let the outcome of an experiment be denoted by the set $OUTCOME$ where the ultimate objective is that any outcome is equally likely. For example consider the flip of a coin.

We model the outcomes by the free type:

$$OUTCOME ::= Head \mid Tail$$

In order to model probability aspects of this specification consider the set $SEED$ where a seed is drawn from some non-empty finite set of integers whose size is a multiple of the number of possible outcomes. For example a seed might be a 64 bit number representing a time.

$$\frac{}{SEED : \mathbb{F}_1 Z} \quad \#SEED \bmod \#OUTCOME = 0$$

The following states that $pick$ makes a fair choice of an outcome given an input seed: the predicate says that $pick$ divides the input sample space evenly amongst the possible outcomes.

$$\frac{pick : SEED \rightarrow OUTCOME}{\forall \# : OUTCOME \bullet \#(pick \sim \{\#\}) = \#SEED \text{ div } \#OUTCOME}$$

The schema $Flip$ models the experiment:

$$\frac{Flip}{\begin{array}{l} seed? : SEED \\ outcome! : OUTCOME \\ \hline outcome! = pick(seed?) \end{array}}$$