FACS - UNIT 19: Exercises

1. Consider the following spec:

SPEC Natural

SORT nat

OPS

zero : -> nat

succ : nat -> nat

add : nat nat -> nat

for all m, n : nat

(1) add(zero, n) = n

(2) add(succ(m), n) = succ(add(m, n))

ENDSPEC

1. Use the signature to generate the numbers 2, 3, 5
2. Hence, use the signature to generate the expression 2+3
3. Use the axioms to prove that 2+3 = 5
4. Can you think of some axioms that define times?
5. Consider the following spec:

SPEC QUEUE

 USING Natural + Boolean

 SORT queue

 OPS

 initq : → queue

 putin : queue nat → queue

 reduce : queue → queue

 front : queue → nat

 is-empty? : queue → bool

 is-in? : stact nat → queue

 queue-error : → queue

 nat-error : → nat

 for all q : queue, n : nat

 AXIOMS for is-empty?:

 (1) ??

 (2) ??

 AXIOMS for front:

 (3) front(initq) = nat-error

 (4) front(putin(q,n)) = if q=initq then n else front(q)

 AXIOMS for reduce:

 (5) reduce(init) = queue-error

 (6) reduce(putin(q,n)) = if q=initq then initq else putin(reduce(q),n)

 ENDSPEC

1. Why does this define a “heterogeneous” algebra?
2. Write down 2 different expressions in the queue’s “term algebra” that represents the same queue.
3. An “is\_empty?” operation takes a queue *q* and returns true if *q* is empty (it contains no numbers), and false otherwise. Write down two axioms that specify the behaviour of the “is\_empty?” operation listed in the signature. Do the same for the “is\_in?” operation.
4. Prove that front(putin(reduce(initq,2),3)) = 3