Logic Exercises

1. Consider the following wffs (where 'P','Q','R', '*the_butler_did_it*' and '*the_butler_has_bloody_hands*' are all propositions):

   (a) $(P \Rightarrow Q) \wedge \neg Q$

   (b) $\neg P$

   (c) $(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

   (d) $P \Rightarrow (Q \Rightarrow P)$

   (e) $(P \vee Q) \wedge (P \vee R) \wedge ((P \vee Q) \Rightarrow R)$

   (f) *the_butler_did_it* $\Rightarrow$ *the_butler_has_bloody_hands*

   (g) *the_butler_has_bloody_hands*

   (i) Given an interpretation:

   P = true

   Q = false

   R = false

   evaluate the wffs (a) - (e) using the truth table method.

   (ii) Determine which of the wffs above are valid, invalid or contingent.

   (iii) Show that (b) logically follows from (a).

   (iv) Show that *the_butler_did_it* DOES NOT logically from (f) and (g).

   (v) Show that R logically follows from (e).

2. Try to translate the following into a formal, logical form (e.g. use a propositional variable 'P' to mean 'Andy Capp's in the pub tonight):

   If Andy Capp's in the pub tonight then he's in trouble.
   Andy Capp's in the pub or at the football match tonight.
   Andy Capp's not in trouble.

   Show how Flo might use formal logic to establish Andy's whereabouts!

3. If we assign false to the proposition

   "The moon is made of green cheese"

   then what can you say about the value of the wff:

   "If the moon is made of green cheese then the world is flat" ?

4. If wff1 logically follows from wff2 what can you say about the truth value of

   (wff2 $\Rightarrow$ wff1) ?

Selected answers

Exercise 1

(i) Evaluate the wffs (a) - (e) using the truth table method

(a)

P	Q	$P \Rightarrow Q$	$\neg Q$	$(P \Rightarrow Q) \wedge \neg Q$
T	F	F	T	F

(b)

P	$\neg P$
T	F

(c)

P	Q	$P \Rightarrow Q$	$(\neg P \vee Q)$	$(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$
T	F	F	F	T

(d)

P	Q	$P \Rightarrow (Q \Rightarrow P)$
T	F	T

(e)

P	Q	R	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R) \wedge ((P \vee Q) \Rightarrow R)$
T	F	F	T	T	F

(ii) Of the wffs above, only (c) is valid, all the rest are contingent.

(iii) The truth table below shows that (b) logically follows from (a) since whenever $(P \Rightarrow Q) \wedge \neg Q$ is true then $\neg P$ is true.

P	Q	$P \Rightarrow Q$	$\neg Q$	$(P \Rightarrow Q) \wedge \neg Q$	$\neg P$
T	T	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	T	**T**	**T**

(iv) The truth table below shows that (g) DOES NOT logically follow from (f) and (g) since there exists an interpretation that leads to $A \Rightarrow B$ begin TRUE when A is false.

A : *the_butler_did_it*
B : *the_butler_has_bloody_hands*

A	B	$A \Rightarrow B$
T	T	T
T	F	F
F	T	T
F	**F**	**T**

(v) Show that R logically follows from (e).

P	Q	R	$P \vee Q$	$P \vee R$	$(P \vee Q) \Rightarrow R$	$(P \vee Q) \wedge (P \vee R) \wedge ((P \vee Q) \Rightarrow R)$
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	F
F	F	T	F	T	T	F
F	F	F	F	F	T	F

Exercise 2

Suppose that P, Q and R are as follows:

P : Andy's in the pub
Q : Andy's at the football
R : Andy's in trouble

and from the question we are given:

$(P \Rightarrow R)$: If Andy Capp's in the pub tonight then he's in trouble
$(P \vee Q)$: Andy Capp's in the pub or at the football match tonight
$\neg R$: Andy Capp's not in trouble

To find out his whereabouts we find the interpretation of P or Q that leads to TRUE for the conjunction of:

$(P \Rightarrow R) \wedge (P \vee Q) \wedge \neg R$

and from the truth table below the only interpretation where it is false has P FALSE and Q TRUE – so Andy is at the football.

P	Q	R	$P \Rightarrow R$	$P \vee Q$	$\neg R$	$(P \Rightarrow R) \wedge (P \vee Q) \wedge \neg R$
T	T	T	T	T	F	F
T	T	F	F	T	T	F
T	F	T	T	T	F	F
T	F	F	F	T	T	F
F	T	T	T	T	F	F
F	T	F	**T**	**T**	**T**	**T**
F	F	T	T	F	F	F
F	F	F	T	F	T	F

Exercise 3

A : The moon is made of green cheese
B : The world is flat
$A \Rightarrow B$: If the moon is made of green cheese then then the world is flat

If A is FALSE then the value of the wff is true – the world is flat !

Exercise 4

If wff1 logically follows from wff2 what can you say about the truth value of (wff2 $\Rightarrow$ wff1) ?

Since wff1 logically follows from wff2 (every interpretation that makes wff2 TRUE makes wff1 TRUE) there is no situation when wff2 is TRUE and wff1 is FALSE. This means that

(wff2 $\Rightarrow$ wff1)

is VALID since every interpretation leads to TRUE.

$wff\,2$	$wff\,1$	$wff\,2 \Rightarrow wff\,1$
T	T	T
F	T	T
F	F	T